Econometric Model for the Formation of the Optimal Structure of the Portfolio of Government Securities of Commercial Banks in the Republic of Moldova

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Abstract: State securities are one of the most attractive instruments both from the point of view of the issuer (government) and from the point of view of the investor.

The aim and objectives of the research is to develop the economic and mathematical model for the formation of an optimal portfolio of government securities in the context of the existence of risk of default and alternative investment preferences.

The material and methodological basis of the research was the works of domestic and foreign economists and practitioners dealing with the problems of government securities portfolio management and investment analysis.

Conclusion: taking into account the situation on the government securities market in the Republic of Moldova, special attention is paid to the risk of default, methods for assessing the risk of stopping the flow of payments have been developed. The analysis of the factors influencing the structure of the optimal portfolio was carried out, and the main parameters that most fully meet the goals of the investor were determined. On the basis of the investigated information, a model of the optimal portfolio structure was created.

Keywords: investments, investment activity, state securities, state securities market, risks of default, economic and mathematical model, econometric model.

1. INTRODUCTION

Actuality of the research topic is determined by the problems related to investments in government securities, namely, the formation of an optimal portfolio of government securities. Investment activity on the state securities market is a complex process that requires consideration of many factors in the modeling process. There are many models that solve the main objectives of investors, such as: forecasting the future value of a security, building a portfolio of the most attractive financial instruments, maximizing the return on the portfolio and minimizing risks.

The state securities market in the Republic of Moldova has a number of disadvantages. However, in a developed market economy, government securities have been and remain the most important financial instrument. Therefore, if the state intends to regulate the economy and influence the economic processes in the country, it will do so not only by creating and controlling the implementation of laws, but also as an active participant in the financial field through financial market interventions. This is fully achieved only after obtaining the status: state as a trusted borrower. Only in these conditions will the state securities market start to function at the required level.

The aim of research is based on the development of the econometric model for the management of a portfolio of government securities, taking into account the new investment conditions in national and international financial markets.

It is therefore necessary to consider all possible events that may occur on the financial market. Therefore, investments in government securities are analyzed taking into account the risk of default, the risk of stopping the flow of payments. It is necessary to research various models of forming an optimal portfolio of government securities, depending on the different preferences of investors, in view of the changes that have taken place in the world economy conditioned by the influence of the COVID 19 pandemic and previous government deficiencies in the state securities market.

Researchers of scientists such as F. Black, Braley, Myers, G. Markovitz, J. Tobin, M. Scholes, as well as economists Burenin A.N., Vatnik P.A., Kantorovich L.V., Lvov Y.A., Pervozvansky A.A, Chetyrkin E.M. are dedicated to risky investment issues.

Investors face the problems of assessing the value of assets, which in turn depends on their risk and return. According to research by Brayle, Mayers, a model is maintained in the market: the higher the potential risk, the higher the expected return. G. Markowitz, J. Tobin pointed out that each investor

has his own predictions about market parameters [Markovitz H.M., 1959]. Pervozvansky A.A., Chetyrkin E.M. argues that the market is constantly moving towards a certain assessment of the balance of risk and return on assets. Possible deviations in estimates are primarily related to asymmetric information held by different investors. In well-developed market conditions, new information is quickly reflected in the market value of assets [Sharp W.F., 2003]. Therefore, for such conditions, it is possible to develop a model that optimally describes the relationship between risk and expected return on assets. This model was developed in the mid-1960s. by W. Sharp and J. Lintern and were named the Capital Asset Pricing Model (CAPM).

Therefore, no paper has been published on the issues of hedging investments in government securities through alternative investments in related assets, due to the fact that the issue has arisen very recently.

2. MATERIAL AND METHODS

The process of creating the model was preceded by an in-depth analysis of all internal and external factors affecting the efficiency and risk of portfolio investments in government securities. The methods of the theory of optimal portfolio, theory of risks, statistical methods and theoretical and probabilistic models were used as a research method. Formation of formulas, verification calculations and the formation of the relationship of factors and indicators were carried out using Excel on a personal computer.

3. RESULTS AND DISCUSSION

In the securities market, government securities are no exception, there are two main types of financial market participants: first, they are organizations or individuals who act in the interest of the client, as intermediaries; the second type are entities that invest their own funds in securities. Therefore, nothing prevents the first group of people to work on the securities market with both the client's funds and their own financial resources.

In the process of managing an investment portfolio, it has to solve two problems. First, risk determination and projected return on the portfolio [Eeckhoudt, C.Collie, 1995]. To do this, he must find out the client's preferences in terms of risk and profitability parameters, tax regime, investment horizon, estimate transaction costs for portfolio formation and management, determine the expected risk and return on assets selected for inclusion in the portfolio. portfolio, the degree of correlation between their returns. Secondly, to determine the real dynamics of portfolio indicators in the process of managing it and, if necessary, to revise it, in the process of selling and buying assets.

In the process of elaborating the econometric model for the formation of the optimal structure of the portfolio, it is necessary to determine the investment horizon and to select the optimal structure in the hypothesis that the risk conditions are stationary within the time horizon. In the next step, when external conditions change, the portfolio structure needs to be revised. The basis of the proposed mechanism for forming a portfolio of government securities is the model for optimizing the portfolio structure.

Government bonds are short-term discount securities with zero coupon, for which the yield to maturity and the maturity date are known. The portfolio consists of different instruments (government securities with different maturities), as a result the flow of payments received is calculated with a known frequency (week, month). The amount of investment funds is denoted by Io and the time horizon of the investment is denoted by **T**. At times **t** = 1, 2, ..., T the securities will be redeemed at yield V and the financial resources received are reinvested in X proportions - in government securities, **1-X** in foreign currency.

The final fixing of securities and foreign exchange earnings takes place at time **T**. The exchange rate changes with a growth rate noted by **C**.

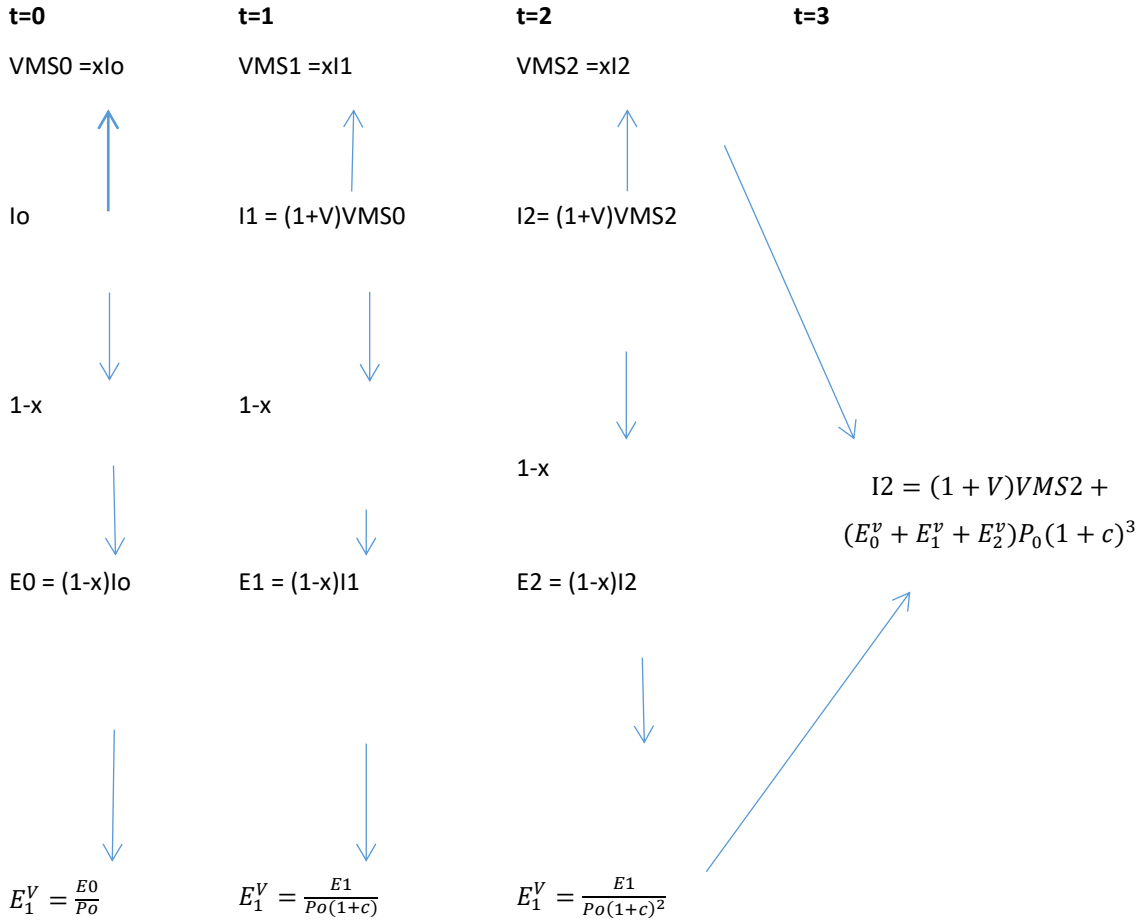
The probability of default (**D**) for the period **t, t + 1** ... is determined based on the estimation of the expected time of default T and takes place mathematical expression: **D** = **1 / (T + 1)**

To determine the T value, the Expert Method can be used, but also data on the dynamics of macroeconomic indicators of the financial market: the ratio between the trend of government loans on the government securities market and the trend of the volume of interbank loans. After the non-fulfillment of obligations, the money previously allocated by investors for the purchase of government securities is transferred to the foreign exchange market, there is a sharp jump in the exchange rate. If bankruptcy and default occurs in the period **t, t + 1**, then the rate of increase of the exchange rate instead of **Ic = 1 + c** will be **Icd = (1 + c) (1 + cd).**

Based on the data on the volume of financial resources that suddenly appeared on the foreign exchange market and the volume of gold and foreign exchange reserves, the value of the cd is

estimated - the increase of the rate in case of bankruptcy (default). **Figure 1** shows a diagram of the distribution process for investments in government securities **VMS** and currency **E** at time **T**.

Figure 1: Diagram of the investment allocation process for T = 3



Source: Developed by the authors

In case of default in the interval **t, t + 1** the income will come only from foreign exchange investments amounting to: $\sum_{t=0}^{t} E_t^V P_0(1+c)^{t+1}(1+cd)$ (1)

The solution to the problem is to build the dependence of the current expected value of income and the investment distribution policy, therefore from x, in the form **V (x)** and the choice **x / max V (x).**

Therefore, the fixing of income on securities takes place at time **T** only in the absence of bankruptcy, the alternative return for these receipts is the rate of increase of the exchange rate **C**. On the other hand, since foreign currency is a market asset with free transactions, the current price is equal to the present value of its future price at any time t under stationary market conditions:

$$P_0 = PV(P_t) = PV[P_0(1+c)^t] \quad (2)$$

Therefore, the present value of a certain amount of foreign currency $\boldsymbol{E^V}$, sold at time t is equal to $\boldsymbol{P_0 E^V}$ or formally with $PV[E^V P_0(1+c)^t] = P_0 E^V$. (3)

Therefore, we will develop the following scheme for calculating the expected value of the revenue stream (Table 1). This diagram describes the probability distribution of possible incomes, where q is the probability value.

Table 1: Scheme for calculating the expected value of the revenue flow

D	$E_0(1+cd)$
Dq	$(E_0+\frac{E_1}{1+c})(1+cd)$
Dq^2	$(E_0+\frac{E_1}{1+c}+\frac{E_2}{(1+c)^2})(1+cd)$
Dq^{T-1}	$(E_0+\frac{E_1}{1+c}+\frac{E_2}{(1+c)^2}\dots+\frac{E_{T-1}}{(1+c)^{T-1}})(1+cd)$
q^T	$E_0+\frac{E_1}{1+c}+\frac{E_2}{(1+c)^2}+\cdots+\frac{E_{T-1}}{(1+c)^{T-1}}+\frac{I_t}{(1+c)^T}$

Source: Developed by the authors

We will note $\frac{q}{1+c}=\frac{1}{1+r}$, where $r=\frac{D+c}{q}$ – expected rate of return (for an investor - neutral risk). Therefore, according to the given probability distribution, the mathematical expectation of today's income is equal to:

$$V_0=E_0+\frac{E_1}{1+c}+\frac{E_2}{(1+c)^2}+\cdots+\frac{E_{T-1}}{(1+c)^{T-1}}+\frac{I_t}{(1+c)^T}+\cdots+cd(1-q^TE_0+(1-q^{T-1})\frac{E_1}{1+c}+\cdots+(1-q)\frac{E_{T-1}}{(1+c)^{T-1}} \quad (4)$$

From the scheme of the process it is observed $E_t=E_0[x(1+V)]^t$, t=1,2…T-1

and $I_t=I_0[x(1+V)]^T$. Here the value x (1 + V) represents the growth rate (index). Therefore, taking into account $E_0=(1-x)I_0$ (5)

Figure 2: Mathematical expectation of today's income

$$V(x)=I_0\begin{Bmatrix}(1-x)\left[1+\frac{x(1+V)}{1+r}+\left(\frac{x(1+V)}{1+r}\right)^2+\cdots+\left(\frac{x(1+V)}{1+r}\right)^{T-1}\right]+\left(\frac{x(1+V)}{1+r}\right)^T+\\ cd(1-x)[(1-q^T)+(1-q^{T-1})\frac{x(1+V)}{1+r}+(1-q^{T-2})\left(\frac{x(1+V)}{1+r}\right)^2+\\ \dots+(1-V)\left(\frac{x(1+V)}{1+r}\right)^{T-1}]\end{Bmatrix}$$

$$=I_0\left[(1-x)\sum_{t=1}^{T}\left(\frac{x(1+V)}{1+r}\right)^{t-1}+\left(\frac{x(1+V)}{1+r}\right)^T+cd(1-x)\sum_{t=1}^{T}(1-q^{T-t+1}\left(\frac{x(1+V)}{1+r}\right)^{t-1}\right]=I_0[H(x)+Jcd(x)] \quad (6)$$

Source: Developed by the authors

Therefore, in the process of developing the model for the formation of an optimal portfolio structure using government securities, it is necessary to conduct research on the following expression:

$$\frac{dV(x)}{dx}=I_0[\frac{dH(x)}{dx}+cd\frac{dJ(x)}{dx} \quad (7)$$

Figure 3: Research of expressions in the process of developing the model for the formation of an optimal portfolio structure using government securities

$$\frac{\mathrm{dH(x)}}{\mathrm{dx}} = -\left[1 + \frac{x(1+V)}{1+r} + \left(\frac{x(1+V)}{1+r}\right)^2 + \cdots + \left(\frac{x(1+V)}{1+r}\right)^{T-1}\right] + \cdots$$

$$+ (1-x)\left[\frac{1+V}{1+r} + 2\left(\frac{1+V}{1+r}\right)^2 + \cdots + (T-1)x^{T-2}\left(\frac{1+V}{1+r}\right)^{T-1}\right] + Tx^{T-1}\left(\frac{1+V}{1+r}\right)^T =$$

$$= \frac{1+V}{1+r}\left[1 + 2\frac{x(1+V)}{1+r} + 3\left(\frac{x(1+V)}{1+r}\right)^2 \ldots + T\left(\frac{x(1+V)}{1+r}\right)^{T-1}\right] - \cdots - [1 + 2\frac{x(1+V)}{1+r}$$

$$+ 3\left(\frac{x(1+V)}{1+r}\right)^2 \ldots + T\left(\frac{x(1+V)}{1+r}\right)^{T-1}]$$

$$J(x) = (1-x)\left[(1-q^T) + (1-q^{T-1})\frac{x(1+V)}{1+r} + (1-q^{T-2})\left(\frac{x(1+V)}{1+r}\right)^2 + \quad \ldots\right.$$

$$\left. + (1-V)\left(\frac{x(1+V)}{1+r}\right)^{T-1}\right]$$

$$= (1-x)\left[1 + \frac{x(1+V)}{1+r} + \left(\frac{x(1+V)}{1+r}\right)^2 + \cdots + \left(\frac{x(1+V)}{1+r}\right)^{T-1} - q^T(1 + \frac{x(1+V)}{1+r}\right.$$

$$\left. + \left(\frac{x(1+V)}{1+r}\right)^2 + \cdots + \left(\frac{x(1+V)}{1+r}\right)^{T-1}\right]$$

Source: Developed by the authors

Figure 4: Mathematical interpretation of expression dh(x)/dx

$$\frac{\mathrm{dH(x)}}{\mathrm{dx}} = -\left[1 + \frac{x(1+V)}{1+r} + \cdots + \left(\frac{x(1+V)}{1+r}\right)^{T-1} - q^T(1 + \frac{x(1+V)}{1+r} + \cdots + \left(\frac{x(1+V)}{1+r}\right)^{T-1}\right] + \cdots$$

$$+ (1-x)\left[\frac{1+V}{1+r} + 2x\left(\frac{1+V}{1+r}\right)^2 + 3x^2\left(\frac{1+V}{1+r}\right)^3 \ldots + (T-1)x^{T-2}\left(\frac{1+V}{1+r}\right)^{T-1}\right.$$

$$\left. - q^T\left(\frac{1+V}{1+r} + 2x\left(\frac{1+V}{1+r}\right)^2 + \cdots + (T-1)x^{T-2}\left(\frac{1+V}{1+r}\right)^{T-1}\right)\right]$$

$$= -\left[1 + \frac{x(1+V)}{1+r} + \ldots + \left(\frac{x(1+V)}{1+r}\right)^{T-1}\right] + q^T\left[1 + \frac{x(1+V)}{1+r} + \ldots + \left(\frac{x(1+V)}{1+r}\right)^{T-1}\right]$$

$$+ \frac{1+V}{1+r}\left[1 + 2x\frac{1+V}{1+r} + 3\left(\frac{(1+V)^2}{1+r}\right)^1 + \cdots + (T-1)x^{T-2}\left(\frac{1+V}{1+r}\right)^{T-2} - \frac{1+V}{1+r}q^T(1\right.$$

$$\left. + 2x\frac{1+V}{1+r} + \ldots + (T-1)\left(\frac{x(1+V)}{1+r}\right)^{T-2}\right]$$

$$- \left[\frac{x(1+V)}{1+r} + 2\left(\frac{x(1+V)}{1+r}\right)^2 + (r-1)\left(\frac{x(1+V)}{1+r}\right)^{T-1}\right]$$

$$+ q^T\left[\frac{x(1+V)}{1+r} + 2\left(\frac{x(1+V)}{1+r}\right)^2 + (T-1)\left(\frac{x(1+V)}{1+r}\right)^{T-1}\right]$$

Source: Developed by the authors

The development of the equations presented in subsequent research will make it possible to form an algorithm for forming the optimal structure of the investment portfolio of commercial banks. This algorithm will make it possible, with a high degree of probability, to determine the investment direction of financial funds for commercial banks, in terms of their distribution between government securities and foreign currency, which in this case presents hedging financial instrument for default risk

4. CONCLUSIONS

The analysis of the development and history of the state securities market in the Republic of Moldova shows that:

1. The market of state securities in the Republic of Moldova is not sufficiently stable.

2. In order to work effectively in the state securities market, it is necessary to be sure of the risk, ie the hedging method is used.

3. It is necessary to develop an effective method of minimizing the risks of investments in the portfolio of government securities.

After analyzing and researching portfolio investments in government securities, we can conclude:

1. Classical models (portfolio theory) are not suitable for a portfolio of government securities under modern conditions.

2. It is necessary to develop a special economic and mathematical model that takes into account the risks of bankruptcy (default), which have a force majeure character.

3. It is necessary to research the model of investment risks with force majeure.

4. In the modern conditions of the Republic of Moldova, the foreign currency can be used as a hedging element or insurance for the risk of bankruptcy.

5. The elaboration of the econometric model and of an algorithm for calculating the structure will allow maximizing the value of the investment portfolio for commercial banks.

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